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Resolvable mating-environmental designs for partial triallel cross experiments

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Abstract

Triallel crosses can be readily exploited as breeding tool for developing commercial hybrids with traits of genetical and commercial importance by acquiring information on specific combining ability effects along with general combining ability effects if the experimentation size is reduced to an economical extent. In this paper, methods of constructing designs involving partial triallel crosses in smaller blocks using different types of lattice designs have been introduced. The designs have low degree of fractionation, which suggests their utility when there is a resource crunch. Canonical efficiency factor of these designs relative to an orthogonal design with same number of lines, assuming constant error variance for both situations, is high indicating that adoption of these designs for the trials could bring about improvement as the recommendations from the experiment will be associated with a high precision.

Key words: Lattice designs, mating-environmental design, partial triallel cross, resolvable, specific combining ability

Introduction

A lot of breeding techniques are used rigorously for the development of hybrids with improved fitness characteristics. All these techniques are based on information collected, on general combining ability (gca) effects and specific combining ability (sca) effects, through experimental designs involving mating plans for choosing best parental lines. The most commonly used mating designs for breeding experiments are diallel crosses which supersedes the use of triallel crosses due to its simplicity and smaller size of experimentation. But, triallel cross experiments can provide more information regarding sca effects, and hybrids based on them are genetically more viable, stable and consistent in performance with stronger buffering mechanism as compared to diallel crosses. If we compare triallel cross with diallel and tetra-allele crosses, we see that triallel crosses are intermediate with respect to selection, ease of testing and resource requirements in terms of crosses. Thus, the only delinquency which deters the breeders to use triallel cross experiments is the huge size of experimentation. Hence, an attempt is made to overcome this situation by developing small and efficient designs for triallel cross experiments.

Triallel cross or complete triallel cross (CTC) has been defined by Rawlings and Cockerham (1962) as a set of all possible three-way matings among a group of lines. In 1965, Hinkelmann defined partial triallel cross (PTC) as a set of triallel matings in which every line occurs r_h and r_f times as half-parent and full-parent, respectively and each cross of the type (i x j) x k {alongwith (i x k) x j and (j x k) x i, to maintain the Structural Symmetry Property (SSP)} occurs either once or not at all. Weatherspoon (1970) recommended usage of triallel crosses as they are more uniform, high yielding and stable than the diallel cross hybrids. Ponnuswamy (1991) gave a method of constructing designs for triallel cross.

Subsequently, triallel crosses find wide application potential in various areas of agriculture and related fields. The silkworm production industry practised triallel crosses for exploitation of heterosis. In a triallel cross experiment conducted by Das et al. (1997), to combine the best characters of multivoltine breed with bivoltine hybrids for tropical conditions, it

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was found that the degree of heterosis varied considerably for different economic characters in different seasons. The genetic investigation of seed size in groundnut was done using triallel experiments to get information on gene action (Varman and Thangavelu 1999). There are many evidences incrops (like maize or corn) and animals (like swine and chicken) where triallel crosses was used for producing commercial hybrids (Shunmuguthai and Srinivasan 2012). Triallel crossbred chickens showed better egg traits and had lower mortalitythan diallel crossbred chickens and (Khawaja et al. 2013). Triallel crossing scheme is very much acceptable and practised in pig farming too.

Harun et al. (2016c) developed some methods of constructing designs involving PTC using Mutually Orthogonal Latin Squares and Partially Balanced Incomplete Block designs. Harun et al. (2016a and b) have developed some methods of constructing designs involving PTC for test vs. control comparisons. More recently, Sharma and Tadesse (2019) have obtained optimal partial triallel cross designs using diallel cross designs. Harun *et al.* (2019) developed PTC plans based on triangular association scheme. Harun et al. (2020) have developed SAS MACRO for generation of PTC design using Triangular Association Scheme.

In this study, we have obtained methods of constructing designs involving partial triallel crosses in smaller blocks using different types of lattice designs. The concept of degree of fractionation is used in this study as it is directly proportional to the resources utilized and designs have low degree of fractionation can be used when there is a resource scarcity. Canonical efficiency factor of the designs is calculated to compare the worth of these designs in comparison to a basic orthogonal design. A design involving partial triallel cross having low degree of fractionation and high canonical efficiency factor is most desirable.

Materials and methods

Consider a triallel cross experiment involving n number of lines giving rise to N number of crosses. Let a cross of type (i x j) x k is represented as (i, j, k).

Model and experimental setup

Full model (Hinkelmann 1965): The following model can be used for representing fixed effect (y_{ijk}) of the triallel cross (i, j, k) :

$$y_{ijk} = \bar{y} + h_i + h_j + g_k + s_{ij} + s_{ik} + s_{jk} + e_{ijk},$$

where $\bar{y}$ is the average effect of the crosses, $\{h_\alpha\}$, $\alpha = i, j$ and g_k represents the gca effects half parents and full parents respectively, $\{s_{\alpha,\beta}\}$, $\{\alpha, \beta\} \in (i, j, k)$ represents the first order sca effects, e_{ijk} represents the second order sca effects, e_{ijk} represents the random error component with the constraints

$$\sum_{i=1}^{n} h_i = 0 \text{ and } \sum_{i=1}^{n} g_i = 0$$

$$\sum_{\alpha\beta} s_{\alpha\beta} = 0 \forall (\alpha,\beta) \in (i, j, k),\ i \neq j \neq k = 1, 2, ..., n \text{ and}$$

$$\sum_{ijk} s_{ijk} = 0 \forall i \neq j \neq k = 1, 2, ..., n.$$

Reduced model (Hinkelmann 1965; Harun et al. 2016a): In this approach, gca effects of first and second kind corresponding to half and full parents will be estimated for which it is assumed that the sca effects are contributing much less to the total combining ability effects as compared to gca effects and hence sca effects are negligible. The model can be written as

$$y_{ijk} = \bar{y} + h_i + h_j + g_k + e_{ijk},$$

where $\bar{y}$ is the average effect of the treatments, $\{h_\alpha\}$, $\alpha = ij$, represents the gca effects of first kind corresponding to the lines occurring as half parents, (g_k) represents the gca effects of second kind corresponding to the lines occurring as full parents, e_{ijk} is the random error component and

$$g_1 + g_2 + \cdots + g_N = 0 \text{ or } \sum_{i=1}^{N} g_i = 0 \text{ and}$$

$$h_1 + h_2 + \cdots + h_N = 0 \text{ or } \sum_{i=1}^{N} h_i = 0.$$

Concepts and definitions

Some terms associated with the proposed designs are explained briefly below:

Degree of fractionation: Let N_{CTC} and N_{PTC} be the number of crosses involved in a CTC and a PTC design, respectively, for a given number of lines n. Then the degree of fractionation f related to designs involving PTC is calculated as:

$$f = \frac{N_{PTC}}{N_{CTC}} = \frac{2N_{PTC}}{n(n-1)(n-2)}.$$

Canonical efficiency factor: Let $\mathbf{C}_{d_{gca-half}}$ and $\mathbf{C}_{d_{gca-full}}$ be the information matrices related to the half parents and full parents, respectively, for agiven design *d*. Let r_h and r_f represent replications of lines as half-parent and full-parent, respectively. Then the canonical efficiency factors pertaining to gca effects of half parents and full parents, E_h and E_f, respectively, is calculated as:

$E_h = \frac{1}{r_h}$ (harmonic mean of non-zero eigenvalues of $\mathbf{C}_{d_{gca-half}}$) and

$E_f = \frac{1}{r_f}$ (harmonic mean of non-zero eigenvalues of $\mathbf{C}_{d_{gca-full}}$).

Resolvable Block Designs: In general a proper block design, is said to be resolvable if its blocks can be grouped into t sets of blocks each containing b/t = x blocks, such that every treatment appears in each set precisely once. Thus these sets of blocks can be taken at a time so as to form groups of blocks, each one of which is a complete replication. Since, triallel cross experiments generally involves huge number of crosses, if a design is resolvable it allows the experimenter to lay out the design in such a manner that each set can be used over different time period or different locations without wasting much resources.

Lattice Designs: Lattice designs belong to the class of incomplete block designs with blocks nested within replications. All types of lattice designs have the property of resolvability *i.e.,* On the basis of structure of treatment number, lattice designs can be mainly classified as Square Lattice, Cubic lattice, Circular lattice and Rectangular lattice.

Square lattice designs (Yates 1940): Square lattice designs have treatment structure of the form $v = s^2$. The parameters of a square lattice design is given as $v = s^2$, $b = is$, $r = i$ and $k = s$, where i = 2, 3, ..., (s + 1). The method of construction of square lattices is based on Mutually Orthogonal Latin Squares (MOLS). For any given treatment structure, the first two replications can be generated by writing the s^2 treatments as a s x s array in the following fashion

$$\begin{vmatrix} 1 & 2 & \cdots & s \\ s+1 & s+2 & \cdots & 2s \\ \vdots & \vdots & \square & \vdots \\ s(s-1)+1 & s(s-2)+2 & \cdots & s^2 \end{vmatrix} \text{ and }$$

$$\begin{vmatrix} 1 & s+1 & \cdots & s(s-1)+1 \\ 2 & s+2 & \cdots & s(s-1)+2 \\ \vdots & \vdots & \square & \vdots \\ s & 2s & \cdots & s^2 \end{vmatrix}$$

The rest of maximum possible (s – 1) replications are generated using MOLS of order s, to constitute a total of (s + 1) replications.

Example 1: Here is an example for $v = s^2 = 9$. The method of construction is based on MOLS of order 3. Writing the 9 treatments in a square array row-wise and column-wise we get six blocks. Then consider the two MOLS of order 3. These two OLS are superimposed on the original 3 x 3 array of the symbols, one by one, and same symbol positions are taken as block contents to get six more blocks. Hence, the four replications are obtained to result in a balanced square lattice design with parameters $v = s^2 = 9$, b = 12, r = 4 and k = 3.

Rep I	Rep II	Rep III	Rep IV
Blk 1 p q r	**Blk 1** p s v	**Blk 1** p u w	**Blk 1** p t x
Blk 2 s t u	**Blk 2** q t w	**Blk 2** q s x	**Blk 2** q u v
Blk 3 v w x	**Blk 3** r u x	**Blk 3** r t v	**Blk 3** r s w
Using rows	**Using columns**	**Using OLS I**	**Using OLS II**

Cubic lattice designs (Das and Giri 1986): Cubic lattices are formed when the treatments can be expressed in the $v = s^3$ structure. Cubic lattice designs belong to the three associate class PBIB designs. These designs have fixed 3 replications and the number of blocks is always a multiple of 3. The parameters of these designs are $v = s^3$, b = $v = s^3$, $v = 3s^3$, r = 3 and $k = s$.

Example 2: Here is an example for = $v = s^3 = 27$. In order to construct the design, first we have to consider the underlying association scheme. First we have to make the following arrangement in which the triplets are the positions of treatments column-wise, block wise and row-wise respectively. The triplet (1 2 3) means that the corresponding treatment 6 is present in the third row of second block in the first column.

1	1	1	1	**10**	2	1	1	**19**	3	1	1
2	1	1	2	**11**	2	1	2	**20**	3	1	2
3	1	1	3	**12**	2	1	3	**21**	3	1	3
4	1	2	1	**13**	2	2	1	**22**	3	2	1
5	1	2	2	**14**	2	2	2	**23**	3	2	2
6	1	2	3	**15**	2	2	3	**24**	3	2	3
7	1	3	1	**16**	2	3	1	**25**	3	3	1
8	1	3	2	**17**	2	3	2	**26**	3	3	2
9	1	3	3	**18**	2	3	3	**27**	3	3	3

Now, two treatments are said to be first associates if they have two positions either of three in common, second associates if they have one position in common, otherwise they are third associates.

The cubic lattice design based on the association scheme with parameters $v = 27$, $b = 27$ $r = 3$ and $k = 3$ is given below

Rep I		Rep II		Rep III	
Blk 1	1 2 3	**Blk 1**	1 4 7	**Blk 1**	1 10 19
Blk 2	4 5 6	**Blk 2**	2 5 8	**Blk 2**	2 11 20
Blk 3	7 8 9	**Blk 3**	3 6 9	**Blk 3**	3 12 21
Blk 4	10 11 12	**Blk 4**	10 13 16	**Blk 4**	4 13 22
Blk 5	13 14 15	**Blk 5**	11 14 17	**Blk 5**	5 14 23
Blk 6	16 17 18	**Blk 6**	12 15 18	**Blk 6**	6 15 24
Blk 7	19 20 21	**Blk 7**	19 22 25	**Blk 7**	7 16 25
Blk 8	22 23 24	**Blk 8**	20 23 26	**Blk 8**	8 17 26
Blk 9	25 26 27	**Blk 9**	21 24 27	**Blk 9**	9 18 27

The blocks of first replication are generated by taking those treatments which are present in the same blocks or treatments having same rows and column positions. The blocks of second replication are constituted by those treatments which are having first and third positions same. The third replication includes the blocks having same second and third position.

Circular lattice designs (Rao 1956): Circular lattices can be formed for treatments which takes the form $v = 2s^2$. Circular lattice designs are three associate class PBIB designs with parameters $v = 2s^2$, $b = 2s$, $r = 2$ and $k = 2s$. The association scheme is established by taking s concentric circles and its s diagonals and the lattice points formed by the intersection of circles and diameters is numbered and considered as treatments. Then, for any treatment the first associates are the treatments on same circle and same diagonal, second associates are the treatments either on same circle or same diagonal, and rest are third associates.

Example 3: Consider an example for $s = 2$ yielding $v = 2s^2 = 8$. Then, we have to take two concentric circles along with the two diagonals as shown in Fig. 3.10.3.

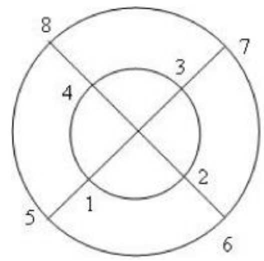

Fig. 1

Now, the circular lattice design can be obtained by developing the blocks of first replication by taking the treatments on one circle as one block. The blocks of second replication are obtained by taking the treatments on one diagonal as one block. Thus, for the given example the design is obtained with parameters $v = 8$, $b = 4$, $r = 2$ and $k = 4$.

Rep-1	**Blk I**	1	2	3	4
	Blk II	5	6	7	8
Rep-2	**Blk I**	1	3	5	7
	Blk II	2	4	6	8

Rectangular lattice designs (Nair 1951): Rectangular lattice designs can be constructed for treatments structure expressed as $v = s(s + 1)$. These designs belongs to the four associate class of PBIB designs with parameters $v = s(s + 1)$ $b = s(s + 1)$, $r = s$ and $k = s$ or $v = s(s + 1)$, $b = (s + 1)^2$, $r = (s + 1)$ and $k = s$.

Rectangular lattice designs with parameters $v = s(s + 1)$, $b = (s + 1)^2$, $r = (s + 1)$ and $k = s$ can be obtained using balanced lattice design for $v = (s + 1)^2$. The method advocates that from the selected balanced lattice design any replication is chosen and deleted and from the rest of replications the extra treatments are discarded to give a rectangular lattice design.

Example 4: Here is an example for $v = s(s + 1) = 6$. Consider the balanced lattice design for 9 treatments. The first replication is deleted and the treatments 7, 8 and 9 are discarded from the rest of replication to give a rectangular design with parameters v = 6, b = 9, r = 3 and k = 2 as shown below:

Rep I	**Blk 1**	1	4
	Blk 2	2	5
	Blk 3	3	6
Rep II	**Blk 1**	1	6
	Blk 2	2	4
	Blk 3	3	5
Rep III	**Blk 1**	1	5
	Blk 2	2	6
	Blk 3	3	4

Method of construction of PTC designs

Consider any existing lattice design (v, b, r, k) with v treatments (expressed as some function ofa positive integer,s) each replicated r times in b blocks each size k each. In general, lattice designs are known to possess blocks of small size. To obtain a design for PTC with n number of lines (= v), from each block all possible triallel crosses are made such that the set of crosses made from all the blocksin a replication of the considered lattice design constitutesa block of the new design.Similarly, all other blocks of the proposed design can be obtained from the remaining replications. Thus, a PTC design (n, N, b*, k*) can be obtained for n (= v) number of lines, with $N\left(=\frac{vr(k-1)(k-2)}{2}\right)$ crosses arranged in b*(= r) blocks each of size $k*\left(=\frac{v(k-1)(k-2).}{2}\right)$. Four classes of PTC designs have been constructed using different types of lattice designs and this has been explained through appropriate examples.

Remark : It may be noted that along with every cross of the type (i, j, k), crosses of the shown in the design layouts presented in examples.

Class I (Square lattices based PTC designs): Any square lattice design with parameters $v = s^2$, b = s(s + 1), r = (s + 1) and k = s, can be used to obtain PTC designs with parameters $n = s^2$, $N=\frac{s^2(s^2-1)(s-2)}{2}$, b* = (s + 1) and $k*=\frac{s^2(s-1)(s-2}{2}$.

Class II (Rectangular lattices based PTC designs): Any rectangular lattice design with parameters v = s(s – 1), $b = s^2$, r = s and k = (s – 1), can be used to obtain PTC designs with parameters, n = s(s – 1), $N=\frac{s^2(s-1)(s-2)(s-3)}{2}$, b* = s, and $k*=\frac{s^2(s-1)(s-2)(s-3)}{2}$.

Class III (Circular Lattices based PTC designs): Any circular lattice design with parameters $v = 2s^2$, b = 2s, r = 2 and k = 2s can be used to obtain PTC designs with parameters n = s(s – 1), $N=\frac{s^2(s-1)(s-2)(s-3)}{2}$, and $k*=\frac{s(s-1)(s-2)(s-3)}{2}$.

Class IV (Cubic lattices based PTC designs): Any cubic lattice design with parameters $v = s^3$, $b = 3s^2$, r = 3 and k = s can be used to obtain PTC designs with parameters $n = s^2$, $N=\frac{3s^3(s-1)(s-2)}{2}$, b* = 3 and $k*=\frac{s^3(s-1)(s-2)}{2}$.

It can be seen that each class of designs has a particular structure for number of lines, but all classes together constitute designs for a wide range of parametric combinations. A program (available with authors) has been written using SAS software [PROC IML] to compute the efficiency factors of designs and variance factors $\{(\bar{V}_{half}(h_i-h_j)$ and $(\bar{V}_{full}(g_i-g_j)\}$ of estimated contrasts pertaining to gca effects, of half as well as full parents.

Results and discussion

Designs under Class I have been constructed for varied number of lines. An example is illustrated here for s = 3 to construct a PTC design for 9 lines by considering a square lattice design (9, 12, 4, 3). Four replications of the lattice design form four blocks of the PTC design (9, 36, 4, 9) as:

Rep 1	**Blk 1**	**Blk 2**	**Blk 3**
	1,2,3	4,5,6	7,8,9
Rep 2	**Blk 1**	**Blk 2**	**Blk 3**
	1,4,7	2,5,8	3,6,9
Rep 3	**Blk 1**	**Blk 2**	**Blk 3**
	1,5,9	3,4,8	2,6,7
Rep 4	**Blk 1**	**Blk 2**	**Blk 3**
	1,6,8	2,4,9	3,5,7
	Square lattice design		

Blk 1	(1×2)×3 (4×5)×6 (7×8)×9
Blk 2	(1×4)×7 (2×5)×8 (3×6)×9
Blk 3	(1×5)×9 (3×4)×8 (2×6)×7
Blk 4	(1×6)×8 (2×4)×9 (3×5)×7
PTC design	

Designs for different values of n have been constructed in Class IItoo. Here, an example is given for a PTC design (12, 48, 4, 12) constructed based on the rectangular lattice design (12, 16, 4, 3):

Rectangular lattice design					
	Rep 1	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,5,9	2,6,10	3,7,11	4,8,12
	Rep 2	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,6,11	2,5,12	3,8,9	4,7,10
	Rep 3	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,8,10	4,5,11	2,7,9	3,6,12
	Rep 4	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,7,12	3,5,10	4,6,9	2,8,11

Now, triallel crosses are made within blocks of each replication to construct a PTC design as:

PTC design					
	Blk 1	(1×5)×9	(2×6)×10	(3×7)×11	(4×8)×12
	Blk 2	(1×6)×11	(2×5)×12	(3×8)×9	(4×7)×10
	Blk 3	(1×8)×10	(4×5)×11	(2×7)×9	(3×6)×12
	Blk 4	(1×7)×12	(3×5)×10	(4×6)×9	(2×8)×11

Again, under Class III, a series of designs have been constructed for different number of lines. For illustration, an example is shown to construct the PTC design (8, 48, 2, 24) based on the circular lattice design (8, 4, 2, 4, 2). All possible triallel crosses are made within each block of a replication to yield a PTC design as given below:

Circular lattice design			**PTC design**	
Rep 1	**Blk 1**	**Blk 2**		(1×2)×3 (1×2)×4 (1×3)×4 (2×3)×4
	1,2,3,4	5,6,7,8	**Blk 1**	(5×6)×7 (5×6)×8 (5×7)×8 (6×7)×8
Rep 2	**Blk 1**	**Blk 2**		(1×3)×5 (1×3)×7 (1×5)×7 (3×5)×7
	1,3,5,7	2,4,6,8	**Blk 2**	(2×4)×6 (2×4)×8 (2×6)×8 (4×6)×8

In a similar manner, designs have also been constructed under Class IV. An example is illustrated here which gives out the PTC design (27, 81, 3, 27) based on the cubic lattice design (27, 27, 3, 3):

Cubic lattice design						
		Blk 1	**Blk 2**	**Blk 3**	**Blk 4**	**Blk 5**
	Rep 1	1,2,3	4,5,6	7,8,9	10,11,12	13,14,15
		Blk 6	**Blk 7**	**Blk 8**	**Blk 9**	
		16,17,18	19,20,21	22,23,24	25,26,27	
		Blk 1	**Blk 2**	**Blk 3**	**Blk 4**	**Blk 5**
	Rep 2	1,4,7	2,5,6	3,6,9	10,13,16	11,14,17
		Blk 6	**Blk 7**	**Blk 8**	**Blk 9**	
		12,15,18	19,22,25	20,23,26	21,24,27	
		Blk 1	**Blk 2**	**Blk 3**	**Blk 4**	**Blk 5**
	Rep 3	1,10,19	2,11,20	3,12,21	4,13,22	5,14,23
		Blk 6	**Blk 7**	**Blk 8**	**Blk 9**	
		6,15,24	7,16,25	8,17,26	9,18,27	

Treating the treatment number as line number, triallel crosses are made within block of each replication of this lattice design to give a PTC design as shown below:

PTC design		
	Blk 1	(1×2)×3 (4×5)×6 (7×8)×9 (10×11)×12 (13×14)×15
		(16×17)×18 (19×20)×21 (22×23)×24 (25×26)×27
	Blk 2	(1×4)×7 (2×5)×6 (3×6)×9 (10×13)×16 (11×14)×17
		(12×15)×18 (19×22)×25 (20×23)×26 (21×24)×27
	Blk 3	(1×10)×19 (2×11)×20 (3×12)×21 (4×13)×22 (5×14)×23
		(6×15)×24 (7×16)×25 (8×17)×26 (9×18)×27

Thus, a series of designs can be obtained for a wide range of parametric combinations through these four methods of construction, but for smaller number of lines with very fewer crosses it may be possible that designs obtained for a given class may be disconnected in the sense that they may not allow the estimation of all pairs of elementary contrasts pertaining to gca effects of half as well as full parents. SAS code has been written under the PROC [IML] section for computing canonical efficiency factor of the design involving triallel crosses for estimating gca effects for half parents as well as full parents under blocked set-up (Supplementary Table S1). Alist of admissible PTC designs (for $n \leq 30$) along with degree of fractionation, variance factors and efficiency factor has been consolidated in Table 1. However, if required one can develop designs using any of the methods for higher number of lines. It can be seen from the

Table 1. List of designs for PTC using lattice designs under blocked setup

n	*b**	*k**	*N*	*f*	$(\bar{V}_{half}(h_i - h_j))$	$(\bar{V}_{full}(g_i - g_j))$	E_h	E_f	Type of lattice design used for construction
8	2	24	48	0.29	0.31	0.47	0.54	0.71	Circular
9	4	9	36	0.14	0.30	0.52	0.84	0.96	Square
16	5	48	240	0.14	0.07	0.14	0.91	0.97	Square
16	4	48	192	0.11	0.11	0.19	0.78	0.89	Square
16	3	48	144	0.09	0.16	0.27	0.69	0.82	Square
16	2	48	96	0.06	0.29	0.46	0.58	0.72	Square
18	2	180	360	0.15	0.06	0.12	0.78	0.87	Circular
20	5	60	300	0.09	0.08	0.15	0.83	0.92	Rectangular
20	4	60	240	0.07	0.11	0.19	0.78	0.89	Rectangular
20	3	60	180	0.05	0.16	0.27	0.69	0.82	Rectangular
20	2	60	120	0.04	0.28	0.47	0.59	0.71	Rectangular
25	6	150	900	0.13	0.03	0.06	0.95	0.99	Square
25	5	150	750	0.11	0.04	0.07	0.91	0.97	Square
25	4	150	600	0.09	0.05	0.09	0.85	0.93	Square
25	3	150	450	0.07	0.07	0.13	0.80	0.89	Square
25	2	150	300	0.04	0.12	0.21	0.72	0.81	Square
30	2	180	360	0.03	0.12	0.21	0.71	0.79	Rectangular

table that all the designs are reasonably efficient with low degree of fractionation. As the number of lines increases, the degree of fractionation decreases and the efficiency factor increases.

Designs for triallel cross experiments find their application in development of plant and animal hybrids but the existing designs, although efficient, require higher number of crosses. Thus, it can be concluded through the results that efficient PTC designs with limitedresources can be used to get information on specific combining ability effects along with general combining ability effects. These designs with lower degree of fractionation can be used advantageously in the conditions of heterogeneous experimental fields where the experimenter has to use scarce or highly valuable resources. Moreover, it is cumbersome task for a breeder to construct and use the designs for triallel crosses based on existing methodologies in literature as it requires theoretical expertise in the domain of statistics. However, the method of construction given in this article is such that it yields combined mating-environmental designs, *i.e.* efficient and economic selection of a fraction of crosses from the complete triallel and laying out these selected sample crosses in blocks for the environmental trial, in a single step.Also, since these designs are constructed from resolvable lattice designs, they are resolvable in terms of lines, which is an additional advantage over the existing designs. This further facilitates the breeders as resolvability ensures the occurrence of each line a constant number of times in each block, besides securing equal replication of the lines in the design.

Authors' contribution

Conceptualization of research (MH, CV, SJ); Designing of the experiments (MH, CV); Contribution of experimental materials (MH); Execution of field/lab/ computational experiments and data collection (EV, CV); Analysis of data and interpretation (EV, CV, MH); Preparation of the manuscript (MH, CV, SJ, EV).

Declaration

The authors declare no conflict of interest.

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Supplementary Table S1. SAS code for computing canonical efficiency factor of the design involving triallel crosses for estimating gca effects for half parents as well as full parents under blocked set-up

```
%let r1=6;/*replication of half parents*/
%let r2=3;/*replication of full parents*/
dataTriallel;
input Block line1 line2 line3;
cards;
1 1 2 3
1 4 5 6
1 7 8 9
1 10 11 12
1 13 14 15
1 16 17 18
1 19 20 21
1 22 23 24
1 25 26 27
1 1 3 2
;
run;
prociml;
usetriallel;
read all into xx;
/*print xx;*/
cross=xx[ ,2]||xx[ ,3]||xx[ ,4];
m=j(nrow(cross),1,1);
/*print cross;*/
x=j(nrow(cross),max(cross),0);
k=1;
doi=1tonrow(cross);
do j=1toncol(cross)-1;
if cross[i,j]>0then
x[k,cross[i,j]]=1;
```

```
end;
k=k+1;
end;
z=j(nrow(cross),max(cross),0);
k=1;
doi=1tonrow(cross);
if cross[i,3]>0then
z[k,cross[i,3]]=1;
k=k+1;
end;
block=j(nrow(xx[ ,1]),max(xx[ ,1]),0);
k=1;
doi=1tonrow(xx[ ,1]);
if xx[i,1]>0then
block[k,xx[i,1]]=1;
k=k+1;
end;
x2=m||block;
c11=(x‘*x)-(x‘*x2)*ginv(x2‘*x2)*(x2‘*x);
c12=(x‘*z)-(x‘*x2)*ginv(x2‘*x2)*(x2‘*z);
c22=(z‘*z)-(z‘*x2)*ginv(x2‘*x2)*(x2‘*z);
c_mat=(c11||c12)//(c12‘||c22);
c_halfparent=c11-c12*ginv(c22)*c12‘;
c_fullparent=c22-c12‘*ginv(c11)*c12;
l=nrow(c_halfparent);
ll=comb(nrow(c_halfparent),2);
contrast=j(ll,l,0);
k=1;
doi=1to l-1;
do j=ito l-1;
contrast[k,i]=1;
```

```
contrast[k,j+1]=-1;
k=k+1;
end;
end;
ginv_hp=ginv(c_halfparent);
ginv_fp=ginv(c_fullparent);
varcov_halfparent=contrast*ginv(c_halfparent)*contrast‘;
varcov_fullparent=contrast*ginv(c_fullparent)*contrast‘;
var_halfparent=j(ll,1,0);
doi= 1toll;
var_halfparent[i,1]=varcov_halfparent[i,i];
end;
ave_var_halfparent=var_halfparent[+, ]/nrow(var_halfparent);
var_fullparent=j(ll,1,0);
doi= 1toll;
var_fullparent[i,1]=varcov_fullparent[i,i];
end;
ave_var_fullparent=var_fullparent[+, ]/nrow(var_fullparent);
eigH=eigval(c_halfparent);
printeigH;
eigF=eigval(c_fullparent);
printeigF;
eigH1=eigH[loc(eigH>0.0000001),];/*positive eigen values*/
eigF1=eigF[loc(eigF>0.0000001),];/*positive eigen values*/
eigH2=eigH1/&r1;
eigF2=eigF1/&r2;
eigH3=1/eigH2;
eigF3=1/eigF2;
CanEffFacH=nrow(eigH3)/sum(eigH3);
CanEffFacF=nrow(eigF3)/sum(eigF3);
printCanEffFacH;
printCanEffFacF;

quit;
```