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Augmented partial diallel cross plans involving two sets of parental lines

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Abstract

Augmented Partial Diallel Cross (APDC) plans are suitable for situations in which resources are limited where a complete diallel is not feasible, but some lines are believed to be superior and are therefore of prime interest. It is desirable to have these primary lines represent a high proportion of crosses and hence they are crossed with every other lines, but a partial diallel system is used for lines of secondary interest. Here, some classes of APDC plans have been obtained using association schemes of Partially Balanced Incomplete Block (PBIB) designs. Variances pertaining to different groups of interline comparisons have been computed and a list consisting of parameters of plans for useful range has been prepared.

Key words: Association schemes, complete diallel cross, general combining ability, partially balanced incomplete block designs, partial diallel cross

Introduction

To improve the genetic potential of plants and animals, breeders generally require information regarding the methods to evaluate the general combining ability (gca) effects of the individual parental lines and the specific combining ability (sca) effects of the various crosses. Mating plans *viz.*, Complete Diallel Cross (CDC) and Partial Diallel Cross (PDC) are very commonly employed to study the combining ability effects of parental lines. A lot of work has been done on various aspects of diallel and partial diallel cross plans. For basic details on theory and analysis of diallel crosses, one may refer to Kempthorne (1956) and, Kempthorne and Curnow (1961). Mating plans for test line versus control line comparison have also been studied by several authors (Das 2003; Hsu and Ting 2005; Srivastava et al. 2013). In some experimental situations, an experimenter may have two groups of parental lines, one group containing lines that are of primary importance and the other group containing lines of secondary interest. It may be always desirable to have superior or well-adapted lines represented in a high proportion of crosses in a breeding programme. To attain this, every primary line is to be crossed with remaining other lines, *i.e.*, a CDC is done while a PDC is done among secondary lines. Mating designs for such situations are called Augmented Partial Diallel Cross (APDC) which is a combination of CDC and PDC. This is used in situations where a CDC is not feasible, and more information is to be obtained about primary lines than about secondary lines in the experiment.

Pederson **(**1980) designed APDC for estimating gca and sca effects of lines. Narain (1990) described method of constructing and analysing APDC plans obtained through circulant plans. Jaggi and Agarwal (1995) developed a systematic analysis of APDC plans. It is observed that there are four types of variances for gca effects whereas for comparing sca effects there are large number of variances indicating that the design is totally unbalanced for sca comparisons. Jaggi and Shukla (1996) made a comparison between APDC and CDC plans.

Association schemes of Partially Balanced Incomplete Block (PBIB) designs can be advantageously used for obtaining APDC plans. PBIB designs are a class of binary, equireplicate and proper incomplete block designs introduced by Bose and Nair (1939). These designs are based on the concept of

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association schemes. General definitions of association scheme and PBIB design are as follows:

An abstract relationship defined on v symbols (lines) is called an m-class association scheme ($m \geq 2$) if the following conditions are satisfied:

(i) Any two lines α and β are either 1^{st}, 2^{nd}, … , or, m^{th} associates, the relation of association being symmetrical, *i.e.*, if α is the i^{th} associate of β, then so is β of α.

(ii) Given a line α, number of lines that are i^{th} associates of α is n_i for i = 1, 2, … , m, where the number n_i does not depend on the lines chosen, *viz.*, α.

(iii) Given a pair of lines α and β, which are mutually i^{th} associates, the number of lines which are simultaneously j^{th} associate of α and k^{th} associate of β is p^i_{jk}, where p^i_{jk} does not depend on the pair of i^{th} associates chosen, *viz.*, α and β.

The integers v, n_i, p^i_{jk} (i, j, k = 1, 2, … , m) are called the parameters of the m-class association scheme.

A PBIB design based on m class ($m \geq 2$) association scheme can be defined, if it is possible to arrange the v lines in b blocks, such that

(i) each block contains k (< v) distinct lines, (ii) each line occurs in r blocks and (iii) if the lines α and â are mutually i^{th} associates in the association scheme, then α and β occur together in λ_i blocks, where the integer λ_i does not depend on the pair (α, β) so long as they are mutually i^{th} associates, i = 1, 2, …, m. Further, not all $ë_i$'s are equal.

The integers v, b, r, k, λ_i are called the parameters of the PBIB design.

In these designs, variance of every estimated elementary contrasts for comparing lines is not the same. A large number of PBIB designs are available in literature.

Here, a general method of construction of APDC plans using association schemes of PBIB designs is given. A program was written in PROC IML of SAS software (2011) to compute variance pertaining to different groups of interline comparisons.

Materials and methods

Let the number of primary and secondary lines be p and q respectively, with p + q = N. In APDC crossing system considered, each primary line is crossed with every other line giving rise to (N – 1) crosses per primary line. Further, a PDC is carried out among secondary lines using association schemes of an m-associate class PBIB design. Crosses between each secondary line with its n_i, i^{th} associates give rise to qn_i (i = 1, 2, … , m) crosses. So there are $(p + n_i)$ crosses per secondary line. Therefore, total number of crosses for APDC plan (N_{APDC}), excluding reciprocals, is $[p(N – 1) + q(p + n_i)]/2$. In another way, N_{APDC} falls into three categories, *i.e.*, crosses among primary lines are p(p-1)/2, crosses of primary lines with secondary lines are pq and PDC among secondary lines are $qn_i/2$. Hence, total number of crosses for APDC plan (N_{APDC}), excluding reciprocals, is $[p(p–1)/2 + pq + qn_i/2] = [p(N – 1) + q(p + n_i)]/2$.

A model for APDC plans is given by

$$y_{ij} = \mu + g_i + g_j + \varepsilon_{ij}, \quad (i, j = 1, 2, ..., N; i \neq j)$$

where, μ is general mean effect, g_i and g_j are gca of line i and j, respectively and ε_{ij} is error component.

Following two- and three-class association schemes (Raghavarao 1960b, 1971; Raghavarao and Chandrasekhararao 1964; Roy 1953; Saha et al. 1974) have been used for constructing APDC plans for a suitable range of parameters:

Two-class association schemes

APDC using triangular association scheme

In triangular association scheme, q = n(n-1)/2 lines ($n \geq 5$) are arranged in a square array of side n, such that the positions on the principal diagonal of the array are left blank, the n(n–1)/2 positions above the principal diagonal are filled up by the q line symbols and positions below the principal diagonal are filled up by the q symbols in such a manner that the resultant arrangement is symmetrical about the principal diagonal. Two lines are first associates if they belong to the same row or same column of the array and are second associates, otherwise. The method of constructing APDC plans for q = 10 lines (n = 5), has been illustrated in the example given below.

Illustration: APDC using triangular association scheme

Let there be 2 primary lines (1 and 2) and 10 (3, 4, 5, 6, 7, 8, 9, 10, 11, 12) secondary lines. One of the possible arrangements of secondary lines on a triangular association scheme and first and second associates of various lines with the association rule that lines falling on same row or column are first associates and others are second associates are given below:

Association Scheme				
*	3	4	5	6
3	*	7	8	9
4	7	*	10	11
5	8	10	*	12
6	9	11	12	*

Lines	First Associates	Second Associates
3	4, 5, 6, 7, 8, 9	10, 11, 12
4	3, 5, 6, 7, 10, 11	8, 9, 12
5	3, 4, 6, 8, 10, 12	7, 9, 11
6	3, 4, 5, 9, 11, 12	7, 8, 10
7	3, 8, 9, 4, 10, 11	5, 6, 12
8	3, 7, 9, 5, 10, 12	4, 6, 11
9	3, 7, 8, 6, 11, 12	4, 5, 10
10	4, 7, 11, 5, 8, 12	3, 6, 9
11	4, 7, 10, 6, 9, 12	3, 5, 8
12	5, 8, 10, 6, 9, 11	3, 4, 7

Both primary lines are crossed with every other line once to give 21 crosses. Further, each secondary line is crossed with its first associates once to give 30 crosses resulting in 51 crosses (crosses to be made are indicated by ×) in the final APDC plan as shown below:

	Lines												
		1	2	3	4	5	6	7	8	9	10	11	12
Lines	1	-	×	×	×	×	×	×	×	×	×	×	×
	2		-	×	×	×	×	×	×	×	×	×	×
	3			-	×	×	×	×	×	×			
	4				-	×	×	×			×	×	
	5					-	×		×		×		×
	6						-			×		×	×
	7							-	×	×	×	×	
	8								-	×	×		×
	9									-		×	×
	10										-	×	×
	11											-	×
	12												-

If a CDC plan with 12 parental lines is used instead of the above APDC plan, 66 crosses are required. In a similar manner, we get another APDC plan using second associates in 36 crosses.

APDC plan using Group Divisible (GD) association scheme

GD association scheme has been used to obtain crosses among secondary lines. In this scheme, q = ls lines (l, s integers; l $\geq$ 2, s $\geq$ 2) are arranged in a rectangular array with l rows and s columns. Two lines are first associates if they belong to the same row of the array and are second associates otherwise.

For l = 2, s = 3, the arrangement of q = 6 (= 2 × 3) lines and APDC plan [p = 5 (1, 2, 3, 4, 5) and q = 6 (6, 7, 8, 9, 10, 11)] constructed using second associates are as shown below:

Association Scheme					
				Associates	
			Lines	First	Second
6	7	8	6	7, 8	9, 10, 11
9	10	11	7	6, 8	9, 10, 11
			8	6, 7	9, 10, 11
			9	10, 11	6, 7, 8
			10	9, 11	6, 7, 8
			11	9, 10	6, 7, 8

	Lines											
		1	2	3	4	5	6	7	8	9	10	11
Lines	1	-	×	×	×	×	×	×	×	×	×	×
	2		-	×	×	×	×	×	×	×	×	×
	3			-	×	×	×	×	×	×	×	×
	4				-	×	×	×	×	×	×	×
	5					-	×	×	×	×	×	×
	6						-			×	×	×
	7							-		×	×	×
	8								-	×	×	×
	9									-		
	10										-	
	11											-

APDC plan using Latin Square (LS) association scheme

Here, crosses among secondary lines of the APDC plans have been obtained using LS association scheme. In a LS association scheme, if q = n^2 lines are arranged in an n × n array then, then two lines appearing in the same row or column are first associates and the others are second associates.

For n = 3, the L_2 association scheme on q = 9 lines and APDC plan [p = 5 (1, 2, 3, 4, 5) and q = 9 (6, 7, 8, 9, 10, 11, 12, 13, 14)] constructed using first associates are as shown below:

Association Scheme					
				Associates	
			Lines	First	Second
6	7	8	6	7, 8, 9, 12	10, 11, 13, 14
9	10	11	7	6, 8, 10, 13	9, 11, 12, 14
12	13	14	8	6, 7, 11, 14	9, 10, 12, 13
			9	6, 10, 11, 12	7, 8, 13, 14
			10	7, 9, 11, 13	6, 8, 12, 14
			11	8, 9, 10, 14	6, 7, 12, 13
			12	6, 9, 13, 14	7, 8, 10, 11
			13	7, 10, 12, 14	6, 8, 9, 11
			14	8, 11, 12, 13	6, 7, 9, 10

	Lines														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Lines	1	-	×	×	×	×	×	×	×	×	×	×	×	×	×
	2		-	×	×	×	×	×	×	×	×	×	×	×	×
	3			-	×	×	×	×	×	×	×	×	×	×	×
	4				-	×	×	×	×	×	×	×	×	×	×
	5					-	×	×	×	×	×	×	×	×	×
	6						-	×	×	×			×		
	7							-	×		×			×	
	8								-			×			×
	9									-	×	×	×		
	10										-	×		×	
	11											-			×
	12												-	×	×
	13													-	×
	14														-

Three-class association schemes

APDC plan using cubic association scheme

Cubic association scheme has been used to obtain crosses among secondary lines. Let there be q = n^3

lines denoted by (α, β, γ), $\alpha, \beta, \gamma = 0, 1, \ldots, (n-1)$. Define the distance d between two lines (α, β, γ) and $(\alpha', \beta', \gamma')$ to be the number of non-null elements in $(\alpha-\alpha', \beta-\beta', \gamma-\gamma')$. Two lines are 1st, 2nd or 3rd associates according as $\delta = 1, 2$, or 3 respectively.

In the example, for n = 2, $q = n^3 = 8$, the eight triplets, association scheme and APDC plan [p = 5 (1, 2, 3, 4, 5) and q = 8 (6, 7, 8, 9, 10, 11, 12, 13)] constructed using first associates are as shown below:

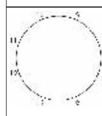
Association Scheme							
	Triplets				Associates		
Lines	α	β	γ		First	Second	Third
6	0	0	0		7, 8, 10	9, 11, 12	13
7	0	0	1		6, 9, 11	8, 10, 13	12
8	0	1	0		6, 9, 12	7, 10, 13	11
9	0	1	1		7, 8, 13	6, 11, 12	10
10	1	0	0		6, 11, 12	7, 8, 13	9
11	1	0	1		7, 10, 13	6, 9, 12	8
12	1	1	0		8, 10, 13	6, 9, 11	7
13	1	1	1		9, 11, 12	7, 8, 10	6

Lines	1	2	3	4	5	6	7	8	9	10	11	12	13
1	-	×	×	×	×	×	×	×	×	×	×	×	×
2		-	×	×	×	×	×	×	×	×	×	×	×
3			-	×	×	×	×	×	×	×	×	×	×
4				-	×	×	×	×	×	×	×	×	×
5					-	×	×	×	×	×	×	×	×
6						-	×	×		×			
7							-		×		×		
8								-	×			×	
9									-				×
10										-	×	×	
11											-		×
12												-	×
13													-

APDC plan using circular association scheme

Circular association scheme has been used to obtain crosses among secondary lines. This scheme has been defined for m associate classes with q = ns lines arranged in s arcs, each of size n. There are two cases, q = 4n and q = 5n. In the first case, there are 4 disjoint sets of lines corresponding to the 4 arcs each having n symbols ($n \geq 2$) on the circumference of a circle. For any line on an arc, other lines in the same arc are first associates, lines appearing in the immediate left and right arcs are second associates and the lines in the remaining one arc are third associates. In the second case, 5 disjoint sets of lines corresponding to the 5 arcs be arranged on the circumference of a circle each having n symbols ($n \geq 2$). For any treatment on an arc, other lines in the same arc are first associates, lines appearing in the immediate left and right arcs are second associates and the lines in the remaining two arcs are third associates.

For n = 2, q = 4n = 8 lines are arranged in the circumference of the circle, and the association scheme and APDC plan [p = 5 (1, 2, 3, 4, 5) and q = 8 (6, 7, 8, 9, 10, 11, 12, 13)] constructed using second associates are as shown below:

Association Scheme		Associates		
	Lines	First	Second	Third
	6	7	8, 9, 12, 13	11, 10
	7	6	8, 9, 12, 13	11, 10
	8	9	6, 7, 10 11	12, 13
	9	8	6, 7, 10, 11	12, 13
	10	11	8, 9, 12, 13	6, 7
	11	10	8, 9, 12, 13	6, 7
	12	13	6, 7, 10, 11	8, 9
	13	12	6, 7, 10, 11	8, 9

Lines	1	2	3	4	5	6	7	8	9	10	11	12	13
1	-	×	×	×	×	×	×	×	×	×	×	×	×
2		-	×	×	×	×	×	×	×	×	×	×	×
3			-	×	×	×	×	×	×	×	×	×	×
4				-	×	×	×	×	×	×	×	×	×
5					-	×	×	×	×	×	×	×	×
6						-		×	×			×	×
7							-	×	×			×	×
8								-		×	×		
9									-	×	×		
10										-		×	×
11											-	×	×
12												-	
13													-

APDC plan using Nested Group Divisible (NGD) association scheme

Crosses among secondary lines are selected using NGD association scheme. Let v = nls (n, l, s = 2) lines be arranged in n groups having l rows and s columns. Two lines are first associates to each other if they belong to same row of the same group, second associates if they occur in different rows of same group and third associates otherwise.

The association scheme for n = 2, l = 2, s = 2 resulting in q = 8 lines (arranged in two groups consisting of 4 lines each in 2 rows and 2 columns) and APDC plan [p = 5 (1, 2, 3, 4, 5) and q = 8 (6, 7, 8, 9, 10, 11, 12, 13)] constructed using third associates are as shown below:

Association Scheme		Associates		
6 7 8 9 10 11 12 13	Lines	First	Second	Third
	6	7	8, 9	10, 11, 12, 13
	7	6	8, 9	10, 11, 12, 13
	8	9	6, 7	10, 11, 12, 13
	9	8	6, 7	10, 11, 12, 13
	10	11	12, 13	6, 7, 8, 9
	11	10	12, 13	6, 7, 8, 9
	12	13	10, 11	6, 7, 8, 9
	13	12	10, 11	6, 7, 8, 9

Lines	1	2	3	4	5	6	7	8	9	10	11	12	13
1	-	×	×	×	×	×	×	×	×	×	×	×	×
2		-	×	×	×	×	×	×	×	×	×	×	×
3			-	×	×	×	×	×	×	×	×	×	×
4				-	×	×	×	×	×	×	×	×	×
5					-	×	×	×	×	×	×	×	×
6						-				×	×	×	×
7							-			×	×	×	×
8								-		×	×	×	×
9									-	×	×	×	×
10										-			
11											-		
12												-	
13													-

Variances pertaining to different groups of interline comparisons *i.e.*, V_{pxp} (both are primary lines), V_{pxq} (one is primary line and the other is secondary line), $\overline{V}_{q\times q\_c}$ (average variance of comparisons among secondary lines that are crossed), $\overline{V}_{q\times q\_nc}$ (average variance of comparisons among secondary lines that are not crossed) and efficiency of APDC plans in comparison to a CDC plan have been computed using PROC IML of SAS software, available online as (Supplementary Table 1). A list consisting of parameters of plans along with these variances and efficiencies is available in the online version (Supplementary Table 2).

Results and discussion

The association schemes of PBIB designs have been effectively used for the construction of APDC plans in small number of crosses. Various association schemes are defined for different structures of number of treatments (here, lines) and hence, are suitable for different situations. For example, a GD association scheme is suitable for any number of primary lines, p and when q, the number of secondary lines is of the form q = ls (l, s $\geq$ 2) while a triangular association scheme can be used when q is of the form q = $^nC_2$ (n $\geq$ 5). A LS association scheme exists when q is square of any natural number $\geq$ 3. Similarly, cubic association scheme (q = n^3), three-class circular association scheme (q = 4n or 5n, n $\geq$ 2) and NGD association scheme (q = nls; n, l, s $\geq$ 2) are useful in obtaining APDC plans depending on the number of secondary lines.

Variances of interline comparisons among primary lines have been estimated with maximum precision in all classes of APDC plans constructed and there is an increasing trend in precision with increase in total number of crosses, N_{APDC}.

For fixed values of p and q, efficiency of APDC plans increase as N_{APDC} increases. For example, when p = 3 and q = 10, efficiencies of APDC plans obtained from a circular association scheme (N_{APDC} = 53), triangular association scheme (N_{APDC} = 63) and GD association scheme (N_{APDC} = 73) are 60.54%, 77.72% and 92.78%, respectively. Again, for a given p, q and N_{APDC}, efficiency for APDC plans obtained using two-class association schemes is at least equal or higher than those obtained using three-class association schemes. For example, when p = 3, q = 8 and N_{APDC} = 39, the efficiency for a APDC plan obtained using first associates of a GD association scheme is 64.66% while for a plan obtained using first associates of a cubic association scheme, the computed efficiency is 63.38%.

When there is scarcity of resources where the experimenter cannot afford to have a CDC for the experiment, APDC plans are appropriate and advantageous. Construction of APDC plans using well known association schemes of PBIB designs is easy. These APDC plans more precisely estimate comparisons among primary lines, at the same time use less number of crosses in comparison to a CDC plan. As number of associate classes in the association scheme used for constructing APDC plans increases, variance is expected to have an increasing trend and the number crosses is expected to decrease. Thus, for a large number of secondary lines, one may opt for a higher associate class association scheme for the construction of APDC plans.

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Supplementary Table 1. SAS code for computing variances pertaining to interline comparisons of APDC plans

```
/*Put entry 1 for cross the I*J in the ijth position of the matrix A and rest keep zero*/
%let p=2; /* no of primary lines*/
%let q=10; /*no of secondary lines*/
/* Plan (using first associates of triangular association scheme) */
prociml;
a={0 1 1 1 1 1 1 1 1 1 1 1,
0 0 1 1 1 1 1 1 1 1 1 1,
0 0 0 1 1 1 1 1 1 0 0 0,
0 0 0 0 1 1 1 0 0 1 1 0,
0 0 0 0 0 1 0 1 0 1 0 1,
0 0 0 0 0 0 0 0 1 0 1 1,
0 0 0 0 0 0 0 1 1 1 1 0,
0 0 0 0 0 0 0 0 1 1 0 1,
0 0 0 0 0 0 0 0 0 0 1 1,
0 0 0 0 0 0 0 0 0 0 1 1,
0 0 0 0 0 0 0 0 0 0 0 1,
0 0 0 0 0 0 0 0 0 0 0 0
};
r=nrow(a);
c=ncol(a);
a1=j(r,c,0);/*CDC matrix*/
doi=1 to r;
do j=1 to c;
if j>i then a1[i,j]=1;
end;
end;
/*print a1;*/
b1=sum(a1);/*total crosses in a cdc*/
/*print c;*/
b=sum(a);/*total crosses in apdc*/
/*print b;*/
x=j(b,r,0);/*design matrix*/
k=1;
doi=1 to r;
do j=1 to c;
if a[i,j]=0 then j=j;
else do;
x[k,i]=1;
x[k,j]=1;
k=k+1;
end;
end;
end;
/*print x;*/
xx=x“*x;
y=j(r,1,0);
doi=1 to r;
doi=1 to r;
y[i,1]=xx[i,i];
end;
end;
/*print y;*/
cmat=(x“*x)-(y*y“)/b;
ginvc=ginv(cmat);
/*print cmat;*/
/*print ginvc;*/
/*Contrast*/
co=j(b1,r,0);
k=1;
doi=1 to r;
do j=1 to r;
if a1[i,j]=0 then j=j;
else do;
co[k,i]=1;
co[k,j]=-1;
k=k+1;
end;
end;
end;
/*print co;*/
cov=co*ginvc*co“;
var1=diag(cov);
one=j(b1,1,1);
var_apdc=var1*one;
/*print var_apdc;*/
cross=j(b1,2,0);
k=1;
doi=1 to r;
do j=1 to c;
if a1[i,j]=0 then j=j;
```

```
else do;
cross[k,1]=i;
cross[k,2]=j;
k=k+1;
end;
end;
end;
/*print cross;*/
av_var_apdc=var_apdc[+, ]/nrow(var_apdc);
/*print av_var_apdc;*/
x1=j(b1,r,0);/*design matrix*/
k=1;
doi=1 to r;
do j=1 to c;
if a1[i,j]=0 then j=j;
else do;
x1[k,i]=1;
x1[k,j]=1;
k=k+1;
end;
end;
end;
/*print x1;*/
x2=j(b1,1,1);
cmat1=(x1“*x1)-(x1“*x2)*ginv(x2“*x2)*(x2“*x1);
ginvc1=ginv(cmat1);
/*print cmat1;*/
cov1=co*ginvc1*co“;
/*print cov1;*/
var11=diag(cov1);
one1=j(b1,1,1);
var_cdc=var11*one1;
/*print var_cdc;*/
cross_var=cross||var_apdc||var_cdc;
/*print cross_var;*/
av_var_cdc=var_cdc[+, ]/nrow(var_cdc);
/*print av_var_cdcav_var_apdc;*/
p1=comb(&p,2);/*total no of primary vs primary crosses*/
cop=j(p1,r,0);/*for var_primary*/
k=1;
doi=1 to &p-1;
do j=i+1 to &p;
cop[k,i]=1;
cop[k,j]=-1;
k=k+1;
end;
end;
/*print cop;*/
covp=cop*ginvc*cop“;
varp1=diag(covp);
onep=j(p1,1,1);
var_primary=varp1*onep;
/*print var_primary;*/
n=&p+&q;
ps=0;
doi= 1 to &p;
m=n-i;
ps=ps+m;
end;
psonly=ps-p1;/*total no of primary vs secondary crosses*/
sec=b1-ps;/*total no of secondary vs secondary crosses
*/
sec_c=b-ps;/*total no of secondary vs secondary which
are crossed */
sec_n=b1-b;/*total no of secondary vs secondary which
are not crossed */
cos=j(sec,r,0);/*for var_secondary*/
k=1;
doi=(&p+1) to (&p+&q)-1;
do j=i+1 to (&p+&q);
cos[k,i]=1;
cos[k,j]=-1;
k=k+1;
end;
end;
/*print cos;*/
covs=cos*ginvc*cos“;
vars1=diag(covs);
ones=j(sec,1,1);
var_secondary=vars1*ones;
/*print var_secondary;*/
av_var_prim=var_primary[+, ]/nrow(var_primary);
av_var_sec=var_secondary[+, ]/nrow(var_secondary);
av_var_prim_sec=(sum(var_apdc)-sum(var_secondary)-
sum(var_primary))/psonly;
co11=j(b,r,0);
k=1;
```

```
doi=1 to r;
do j=1 to r;
if a[i,j]=0 then j=j;
else do;
co11[k,i]=1;
co11[k,j]=-1;
k=k+1;
end;
end;
end;
/*print co11;*/
cov11=co11*ginvc*co11";
/*print cov11;*/
varc=diag(cov11);
one=j(b,1,1);
var_c=varc*one;
/*print var_c;*/
av_var_sec_c=(sum(var_c)-sum(var_primary)-(psonly*av_var_prim_sec))/sec_c;
av_var_sec_n=(sum(var_apdc)-sum(var_c))/sec_n;
/*efficiency per cross*/
/*e1=(p1*av_var_prim)/(b1*av_var_cdc);*/
e2=(psonly*av_var_prim_sec)/(b1*av_var_cdc);
e3=(sec_c*av_var_sec_c)/(b1*av_var_cdc);
/*e4=(sec_n*av_var_sec_n)/(b1*av_var_cdc);*/
/*e=(b*av_var_apdc)/(b1*av_var_cdc);*/
print b1 b p1 psonly sec sec_csec_n;
printav_var_primav_var_prim_secav_var_sec_cav_var_sec_nav_var_cdc;
printav_var_apdc;
```

Supplementary Table 2. List consisting of parameters and efficiency of APDC plans

S.No.	p	q	N_{APDC}	N_{CDC}	V_{pxp}	V_{pxq}	$\overline{V}_{q\times q\_c}$	$\overline{V}_{q\times q\_nc}$	Efficiency	Association scheme	Associates used
1	5	8 (=4×2)	74	78	0.1818	0.1917	0.202	0.1818	0.9416	GD	Second
2	4	10 (=5×2)	86	91	0.1667	0.1749	0.1833	0.1667	0.9384	GD	Second
3	3	12 (=6×2)	99	105	0.1538	0.1608	0.1678	0.1538	0.9369	GD	Second
4	5	6 (=3×2)	52	55	0.2222	0.2377	0.254	0.2222	0.9353	GD	Second
5	4	8 (=4×2)	62	66	0.2	0.2122	0.225	0.2	0.9302	GD	Second
6	3	10 (=5×2)	73	78	0.1818	0.1917	0.202	0.1818	0.9278	GD	Second
7	2	12 (=6×2)	85	91	0.1667	0.1749	0.1833	0.1667	0.9269	GD	Second
8	4	6 (=3×2)	42	45	0.25	0.2701	0.2917	0.25	0.9197	GD	Second
9	3	8 (=4×2)	51	55	0.2222	0.2377	0.254	0.2222	0.9151	GD	Second
10	2	10 (=5×2)	61	66	0.2	0.2122	0.225	0.2	0.9139	GD	Second
11	3	6 (=3×2)	33	36	0.2857	0.313	0.3429	0.2857	0.8973	GD	Second
12	2	8 (=4×2)	41	45	0.25	0.2702	0.2917	0.25	0.8947	GD	Second
13	5	9 (=3×3)	82	91	0.1667	0.1849	0.2045	0.1818	0.8846	GD	Second
14	3	12 (=4×3)	93	105	0.1538	0.1691	0.1852	0.1667	0.8708	GD	Second
15	4	9 (=3×3)	69	78	0.1818	0.2042	0.2286	0.2	0.8640	GD	Second
16	2	6 (=3×2)	25	28	0.3333	0.3725	0.4167	0.3333	0.8639	GD	Second
17	5	6 (=2×3)	49	55	0.2222	0.2582	0.3	0.25	0.8627	GD	Second
18	2	12 (=4×3)	79	91	0.1667	0.185	0.2045	0.1818	0.8506	GD	Second
19	3	9 (=3×3)	57	66	0.2	0.228	0.2593	0.2222	0.8377	GD	Second
20	5	10	90	105	0.1538	0.1786	0.2037	0.1852	0.8345	Triangular	First
21	4	6 (=2×3)	39	45	0.25	0.2984	0.3571	0.2857	0.8278	GD	Second
22	4	10	76	91	0.1667	0.1965	0.2273	0.2045	0.8090	Triangular	First
23	5	8 (=2×4)	66	78	0.1818	0.2203	0.2667	0.2222	0.8075	GD	Second

24	5	8 (=4×2)	66	78	0.1818	0.2203	0.2667	0.2222	0.8075	Circular	Second
25	5	8(=2×2×2)	66	78	0.1818	0.2203	0.2667	0.2222	0.8075	NGD	Third
26	2	9 (=3×3)	46	55	0.2222	0.2585	0.3	0.25	0.8028	GD	Second
27	3	12 (=3×4)	87	105	0.1538	0.1793	0.2078	0.1818	0.8012	GD	Second
28	5	6 (=2×3)	46	55	0.2222	0.2824	0.3333	0.2963	0.7957	GD	First
29	3	10	63	78	0.1818	0.2187	0.2571	0.2286	0.7772	Triangular	First
30	3	6 (=2×3)	30	36	0.2857	0.3545	0.4444	0.3333	0.7770	GD	Second
31	2	12 (=3×4)	73	91	0.1667	0.1978	0.2333	0.2	0.7694	GD	Second
32	4	8 (=2×4)	54	66	0.2	0.2496	0.3125	0.25	0.7680	GD	Second
33	4	8 (=4×2)	54	66	0.2	0.2496	0.3125	0.25	0.7680	Circular	Second
34	4	8(=2×2×2)	54	66	0.2	0.2496	0.3125	0.25	0.7680	NGD	Third
35	5	10 (=2×5)	85	105	0.1538	0.1924	0.24	0.2	0.7635	GD	Second
36	5	9	73	91	0.1667	0.2115	0.2571	0.2286	0.7611	LS	First
37	5	9	73	91	0.1667	0.2115	0.2571	0.2286	0.7611	LS	Second
38	5	8	62	78	0.1818	0.2368	0.2857	0.2597	0.7514	Cubic	Second
39	4	6 (=2×3)	36	45	0.25	0.3313	0.4	0.35	0.7499	GD	First
40	5	8 (=2×4)	62	78	0.1818	0.2368	0.2857	0.2597	0.7473	GD	First
41	5	8	62	78	0.1818	0.2386	0.2984	0.2571	0.7442	Cubic	First
42	2	10	51	66	0.2	0.2467	0.2963	0.2593	0.7369	Triangular	First
43	4	10 (=2×5)	71	91	0.1667	0.215	0.2778	0.2222	0.7226	GD	Second
44	4	9	60	78	0.1818	0.2379	0.2963	0.2593	0.7215	LS	First
45	4	9	60	78	0.1818	0.2379	0.2963	0.2593	0.7215	LS	Second
46	5	10 (=2×5)	80	105	0.1538	0.204	0.25	0.2308	0.7146	GD	First
47	3	8 (=2×4)	43	55	0.2222	0.2892	0.381	0.2857	0.7134	GD	Second
48	3	8 (=4×2)	43	55	0.2222	0.2892	0.381	0.2857	0.7134	Circular	Second
49	3	8(=2×2×2)	43	55	0.2222	0.2892	0.381	0.2857	0.7134	NGD	Third
50	5	10 (=5×2)	80	105	0.1538	0.2061	0.2637	0.2282	0.7078	Circular	Second
51	5	10 (=5×2)	80	105	0.1538	0.2061	0.2637	0.2282	0.7078	Circular	Third
52	4	8	50	66	0.2	0.2704	0.3333	0.3	0.7077	Cubic	Second
53	4	8 (=2×4)	50	66	0.2	0.2704	0.3333	0.3	0.7027	GD	First
54	4	8	50	66	0.2	0.2735	0.3542	0.2969	0.6965	Cubic	First
55	2	6 (=2×3)	22	28	0.3333	0.441	0.6	0.4	0.6952	GD	Second
56	5	6 (=3×2)	43	55	0.2222	0.3208	0.4	0.3429	0.6915	GD	First
57	3	6 (=2×3)	27	36	0.2857	0.4011	0.5	0.4286	0.6887	GD	First
58	4	12 (=2×6)	90	120	0.1429	0.189	0.25	0.2	0.6877	GD	Second
59	4	12 (=4×3)	90	120	0.1429	0.189	0.25	0.2	0.6877	Circular	Second
60	4	12(=2×2×3)	90	120	0.1429	0.189	0.25	0.2	0.6877	NGD	Third
61	5	8 (=4×2)	58	78	0.1818	0.2638	0.3429	0.2959	0.6743	Circular	Third
62	5	8(=2×2×2)	58	78	0.1818	0.2638	0.3429	0.2959	0.6743	NGD	Second
63	4	10 (=2×5)	66	91	0.1667	0.2285	0.2857	0.2619	0.6735	GD	First
64	3	9	48	66	0.2	0.2724	0.35	0.3	0.6716	LS	First
65	3	9	48	66	0.2	0.2724	0.35	0.3	0.6716	LS	Second
66	3	10 (=2×5)	58	78	0.1818	0.2448	0.3333	0.25	0.6683	GD	Second

67	4	10 (=5×2)	66	91	0.1667	0.2321	0.3068	0.2591	0.6624	Circular	Second
68	4	10 (=5×2)	66	91	0.1667	0.2321	0.3068	0.2591	0.6624	Circular	Third
69	3	8	39	55	0.2222	0.3153	0.4	0.3556	0.6528	Cubic	Second
70	4	12 (=2×6)	84	120	0.1429	0.198	0.25	0.2321	0.6512	GD	First
71	5	10	75	105	0.1538	0.2243	0.2963	0.2593	0.6468	Triangular	Second
72	3	8 (=2×4)	39	55	0.2222	0.3153	0.4	0.3556	0.6466	GD	First
73	3	12 (=2×6)	75	105	0.1538	0.2125	0.2963	0.2222	0.6349	GD	Second
74	3	12 (=4×3)	75	105	0.1538	0.2125	0.2963	0.2222	0.6349	Circular	Second
75	3	12(=2×2×3)	75	105	0.1538	0.2125	0.2963	0.2222	0.6349	NGD	Third
76	3	8	39	55	0.2222	0.3217	0.4381	0.3524	0.6338	Cubic	First
77	2	15	91	136	0.1333	0.1801	0.2292	0.2083	0.6327	Triangular	First
78	2	8 (=2×4)	33	45	0.25	0.3482	0.5	0.3333	0.6313	GD	Second
79	2	8 (=4×2)	33	45	0.25	0.3482	0.5	0.3333	0.6313	Circular	Second
80	2	8(=2×2×2)	33	45	0.25	0.3482	0.5	0.3333	0.6313	NGD	Third
81	3	10 (=2×5)	53	78	0.1818	0.26	0.3333	0.303	0.6237	GD	First
82	4	6 (=3×2)	33	45	0.25	0.3889	0.5	0.4167	0.6221	GD	First
83	4	8 (=4×2)	46	66	0.2	0.3083	0.4167	0.35	0.6168	Circular	Third
84	4	8(=2×2×2)	46	66	0.2	0.3083	0.4167	0.35	0.6168	NGD	Second
85	5	9 (=3×3)	64	91	0.1667	0.2551	0.3333	0.2963	0.6113	GD	First
86	3	14 (=2×7)	94	136	0.1333	0.1879	0.2667	0.2	0.6093	GD	Second
87	2	9	37	55	0.2222	0.3195	0.4286	0.3571	0.6076	LS	First
88	2	9	37	55	0.2222	0.3195	0.4286	0.3571	0.6076	LS	Second
89	3	12 (=2×6)	69	105	0.1538	0.2213	0.2857	0.2637	0.6067	GD	First
90	3	10 (=5×2)	53	78	0.1818	0.2664	0.3687	0.3005	0.6054	Circular	Second
91	3	10 (=5×2)	53	78	0.1818	0.2664	0.3687	0.3005	0.6054	Circular	Third
92	2	6 (=2×3)	19	28	0.3333	0.5093	0.6667	0.5556	0.6043	GD	First